IMPROVED COMPUTER SOFTWARE FOR THE TEACHING OF ORDINARY DIFFERENTIAL EQUATIONS: The ODEToolkit in the Classroom Setting

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ABSTRACT

We discuss a collection of commonly-accepted goals for future education in the field of ordinary differential equations, especially as involves the uses of technology in the classroom. We single out several of these as particularly in need of further emphasis, and discuss methods for improvement. Specifically, we introduce the ODEToolkit, a free, online differential equations solver designed specifically to help a professor incorporate these paradigms into an introductory course.

1 Introduction

Ordinary differential equations are an extraordinarily useful tool in mathematics. Yet for many students their first experience with them in a classroom setting is a rote application of previously learned concepts. This can lead the student to develop the opinion that ODEs—and possibly advanced math in general—is a mundane and formulaic subject.

All too often instruction in ODEs takes the form of rote memorization. Students are given several formulae and told to place an ODE into the proper category and apply the formula for that category. A classic example of this is the integrating factor approach to solving linear first order ordinary differential equations. Students are given a brief explanation of where this comes from and then have to use it before they truly understand why it works.

This is a problem. Students should not be forcibly dragged through material that is interesting on its own merits. The challenge of teaching ODEs is to make these merits visible. If professors spend more time explaining the *why* instead of emphasizing the memorization of *how*, students will come away from their courses with a much deeper understanding of the mathematics involved.

Consequently we advocate a new approach to teaching differential equations; one that will emphasize the *why* without neglecting the importance of the *how*. This approach is based on the current revolution in teaching brought on by the advent of the personal computer. We take this one step further by taking advantage of the capabilities of the Internet. Specifically we introduce the ODEToolkit, a free online ordinary differential equations solver. This new software makes it easy for professors to implement our approach to teaching differential equations in their classrooms. The remainder of this paper will outline several current theories in teaching ODEs, examine how our approach expands upon these ideas, and how our software performs under these paradigms.

2 Our Approach

As we have mentioned earlier, our approach is based on the commonly accepted idea that differential equations should be much more than mere memorization and application. Many experts have spent a great deal of time designing better and more effective methods of teaching ODEs. As such we will spend the first portion of this section outlining what we believe the most important pieces of these ideas are, and how we have integrated them into our approach. After this, we will present our improvements to these methods.

2.1 Common Ideas

Ever since personal computers have become powerful enough to be useful for mathematics, a great deal of research has gone into using these tools effectively in the classroom. While there are many ideas on how to do this, most of the methods have a few things in common. We have attempted to integrate these common features into our approach. The most important idea is that computers can not replace conventional teaching. They can only augment it. We are not advocating the replacement of classical teaching styles with computer learning. However, when used appropriately, the computer can be a very effective aid in the classroom. One common way this is done is to emphasize a computer modeling approach to teaching ODEs. Instead of having students solve arbitrary equations, they are taught to set up a model of a real life situation and solve the resulting ODE.

For example, consider the case of Jimmy. Jimmy is a second year undergraduate student taking ODEs. He has just learned about first order linear equations and is doing exercises to improve his abilities. Classically he would be given a list of equations to solve, using the same technique over and over until he had mastered it. This is an effective way to learn how to perform a task, but a very ineffective way to teach understanding. Also, it is quite boring. A better way of teaching this concept would be to give Jimmy a real-life situation and ask him to answer real-life questions. A good example can be found in Cooper and LoFaro's paper *Differential Equations on the Internet*. Their example of salmon migration uses the latest data available over the Internet to model the number of young salmon that can migrate past dams on the Snake and Columbia rivers. This is a very interesting problem, and as such would engage Jimmy's mind in a much deeper fashion than simply solving arbitrary equations.

The idea of using modeling to teach differential equations is by no means new. However, it illustrates one method of improving a class by focusing on methods other than the classic "apply the formula" technique.

Another important concept for teaching ODEs is visualization. One way to get a good grasp of how equations behave and what the solutions are is to use a visual approach. Computers are an excellent tool for doing this. Using computer software a student can instantly view the myriad solution curves to a given differential equation. What better way to examine the existence and uniqueness theorems of ODEs than to actually draw the solution curves and examine how they never meet yet still hit every point? This is a very powerful technique, yet without a computer it is virtually impossible to use effectively.

The last commonly acknowledged advantage to using computers that we will speak of is the use of numerical solvers. Without a computer with a numerical solver, models must be limited to those which lead to equations with nice, clean, analytical solutions. Unfortunately, these models tend to be very artificial. By using numerical solvers we remove this restraint and allow models to become much more complex and often more accurate. This in turn leads to a better understanding of the usefulness of ODEs in solving real life problems, something that we have already argued is very important.

2.2 Our improvements

Although these measures are certainly steps in the right direction, we feel that there are aspects of teaching differential equations that need significantly more emphasis than are usually accorded to them. Consequently, we have designed the ODEToolkit to take advantage of the computer revolution to display these properties. The points we wish to emphasize will be detailed in the remainder of this section, after which we will address their application in the ODEToolkit and its potential use in teaching undergraduates.

First and foremost, we feel that the cornerstone to a good education in differential equations is interactivity. It is worth noting that it can certainly be a useful exercise to draw out by hand various solutions to a differential equation. This does not compare, however, to the insight gained by a student from instantly receiving feedback on how her choice of parameters or initial conditions affects the solution curves. It is when a student has at her fingertips the ability to quickly and easily manipulate the relevant data, and only then, that a student can understand how important every aspect of a differential equation (be it its linearity, its initial conditions, its order, etc.) is to its solutions. An example of this is shown in Figure 1. In this example we illustrate the ability to modify parameters quickly and easily, allowing students to view the effects of their changes on the solution curves.

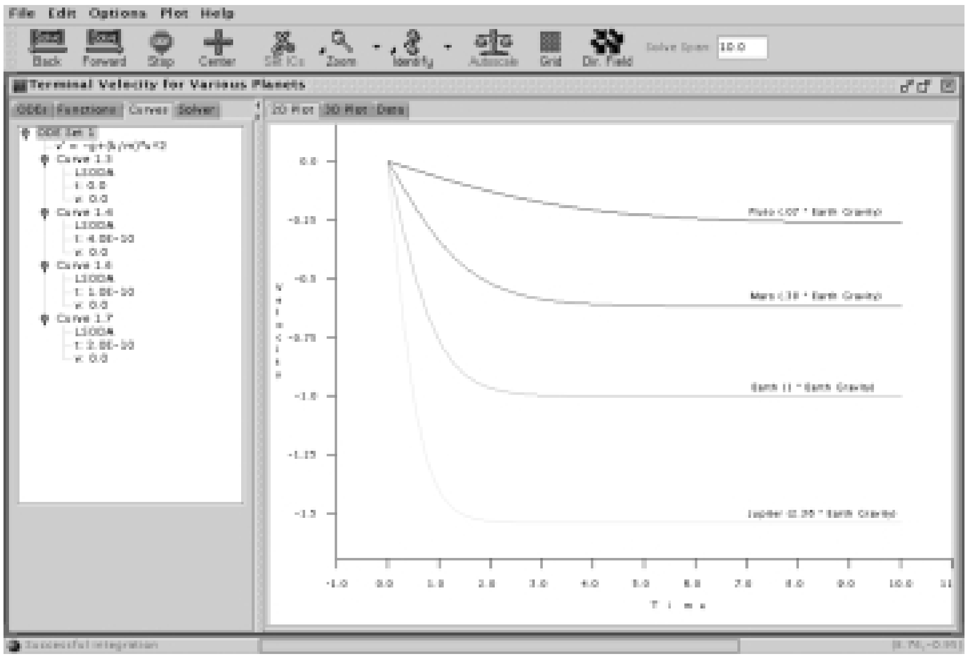

Figure 1: Normalized terminal velocities on various planets, neglecting changes in atmosphere.

A second topic of interest, which goes hand in hand with the previous one, is that of visualization. The ability to interact real-time with solution curves is an important aspect of this, but there are other facets as well. The ability to view solutions from every angle is not only desirable, but often necessary for a deeper understanding of a particular concept. As an example, let us return to the uniqueness theorem of ODEs, as applied to a system of differential equations. As seen in Figure 1 it is quite possible to have solution curves to a system of differential equations appear to intersect on the x-y plane, leading to possible confusion. It is only after viewing the curve from a higher dimensional viewpoint that the student sees the truth; that the two solution curves are actually quite distinct. This is demonstrated in Figure 2.

Another useful teaching tool is to use famous examples. There are literally hundreds of ODEs that are commonly used in real-life applications, and examining these in detail can be very enlightening. Because of this, we feel that an early introduction to a variety of these classic ODEs can drastically improve a student's enjoyment and understanding of the subject. As such, including a library of as many useful ODEs as possible is a very helpful addition to any computer software package. A good example of this is the Autocatalator reaction, a model of an oscillating chemical reaction as seen in Figure 3.

A final aspect in which we feel improvement is necessary is an ever-present problem facing those who wish to integrate computers into the classroom. Specifically, it

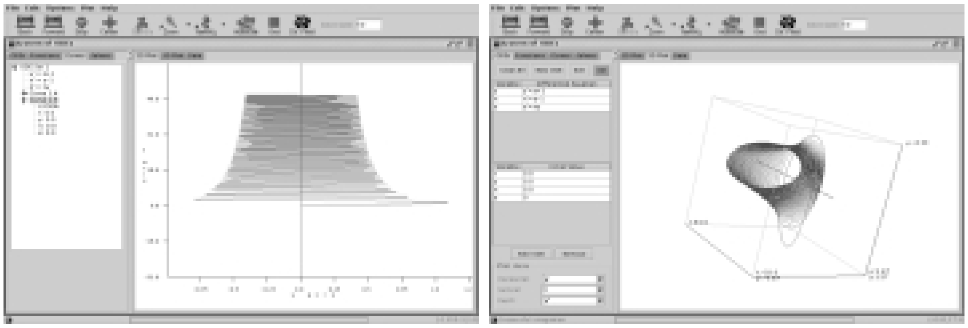

Figure 2: $y' = z + 1, \quad x' = z - 1, \quad z' = xy$. The first picture seems to violate the uniqueness theorem. However, with the proper viewpoint this is shown not to be the case.

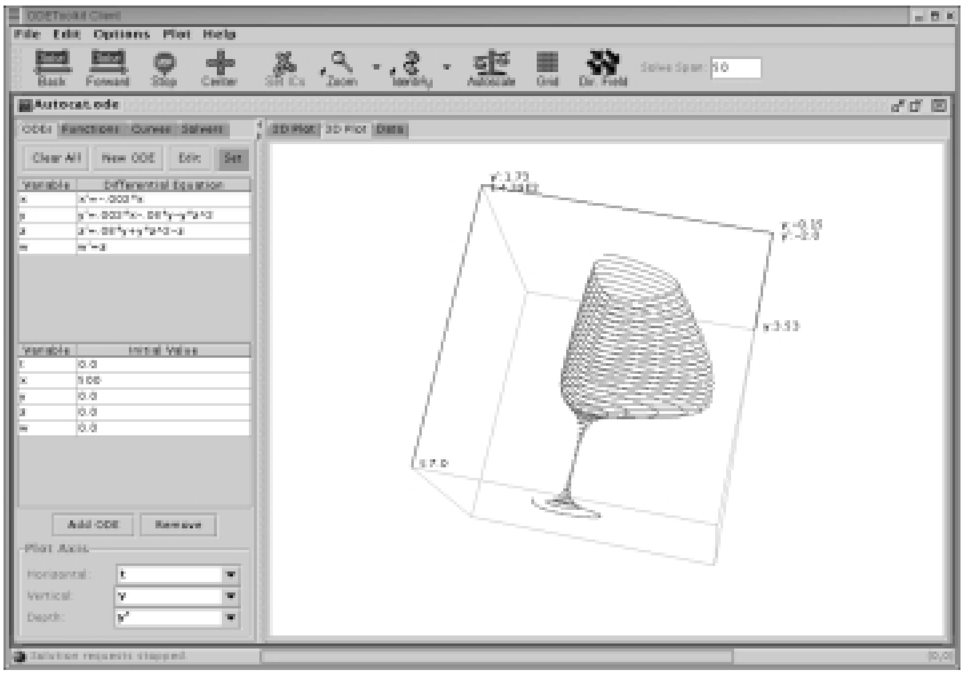

Figure 3: The Autocatalator reaction taken with a solve length of 500.

is difficult to learn through two disconnected media at once. Even though desirable for presenting multiple facets of learning, alternating between the professor's algebraic approach and the computer's graphical approach can be jarring to the student. In addition each switch between these two methods takes valuable time away from the course. Thus, we propose a stronger and more direct link between the education provided by the professor and the insight provided by new technologies.

3 Our Software

The purpose of the ODEToolkit was, and is, to design an interface with which students and connoisseurs alike can interface with differential equations in an educational and informative setting. The program should be equally useful for students in an introductory course, engineers who need solutions to real-life problems, to PhDs curious in the appearance of esoteric solution curves, and to anyone simply interested in learning. Consequently, the aforementioned goals were constantly in mind when designing and upgrading the ODEToolkit. This section will attempt to describe the extent to which our design goals are manifested in the software.

The commonly accepted ideas that we have presented in this paper are all met by our software. Of course, some of them, such as modeling, are not really a function of our software. Many of these ideas require a dedicated and insightful professor at the reins. The ODEToolkit does, however, meet most of the ideas that are dependent solely on the software package. This of course includes our improvements.

Accordingly, we have an excellent visualization package which will allow real time manipulation of a system of ODEs in any 3 dimensions. The solution curves can then be rotated freely, allowing a great deal of insight into the inner works of the ODE. We have also integrated a great deal of interactivity. It is very easy for a student to quickly pick out a multitude of initial conditions to see as many solution curves as she wishes. In addition, we maintain an extensive online library to allow students to view the standard ODEs from physics, chemistry, and engineering as well as the many commonly used ODEs in mathematics. A future goal is to let users enter their own ODEs into the library, allowing researchers to share results from their field of work or students to comment on interesting new equations.

The last of the goals we presented was to reduce the discrepancy between the education a student receives from her professor and the education she receives via technological means. Although still in the early stages of development, one of the key improvements we are planning for the ODEToolkit will help ameliorate this problem. Specifically, we are working to put a separate user interface on the program for owners of the textbook *Differential Equations: A Modeling Perspective*, by Harvey Mudd College Professors Robert Borrelli and Courtney Coleman. This new interface will allow students to work examples and homework problems from the textbook directly from the ODEToolkit. Consequently, we feel that by using the ODEToolkit in conjunction with conventional means the gap between these two methods of teaching will be dramatically diminished.

4 Conclusion

There are several goals to which the modern day differential equations course should strive, many of which derive from our new found ability to introduce computing technology to the classroom. These include possibilities for visualization, and a greater degree of interaction between a student and her mathematical surroundings. The ODEToolkit encapsulates these goals, giving students a large degree of freedom to experiment and interface with a powerful solver. It is our hope that these methods will lead students to a greater level of insight and intuition than is often achieved via classical means.

5 Biography of the ODEToolkit

The ODEToolkit is a powerful online differential equations solver available for free at `http://odetoolkit.hmc.edu`. It currently requires a web browser equipped with a version of Java at least as recent as Java 1.2. The project has been sponsored by the

Mathematics Clinic at Harvey Mudd College, where it is developed and maintained by a team of undergraduate students under the direction of Professors Borrelli, Coleman, and Raugh.

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REFERENCES

-Anton, H. and Trench, W.F. 2001. "Elementary Differential Equations with Boundary Value Problems." Brooks/Cole.

-Borrelli, R. and Coleman, C.S., 1999. "Modeling and Visualization in the Introductory ODE Course", *Revolutions in Differential Equations: Exploring ODEs with Modern Technology*, The Mathematical Association of America, Notes #50. 1-12.

-Borrelli, R.L. and Coleman, C.S. 1998. "Differential Equations: A Modeling Perspective." John Wiley & Sons. New York, New York.

-Borrelli, R. L., Coleman, C. and Boyce, William E., 1992. "Differential Equations Laboratory Workbook." John Wiley & Sons. New York, New York.

-Boyce, William E., 1999. "Differential Equations in the Information Age", *Revolutions in Differential Equations: Exploring ODEs with Modern Technology*, The Mathematical Association of America, Notes #50. 13-18.

-Cooper, Kevin D. and LoFaro, Thomas, 1999. "Differential Equations on the Internet", *Revolutions in Differential Equations: Exploring ODEs with Modern Technology*, The Mathematical Association of America, Notes #50. 39-50.

-West, Beverly H., 1999. "Technology in Differential Equations Courses: My Experiences, Student Reactions", *Revolutions in Differential Equations: Exploring ODEs with Modern Technology*, The Mathematical Association of America, Notes #50. 79-89.

-Zil, Dennis G. 2001. "A First Course in Differential Equations with Modeling Applications." Seventh Edition, Brooks/Cole.